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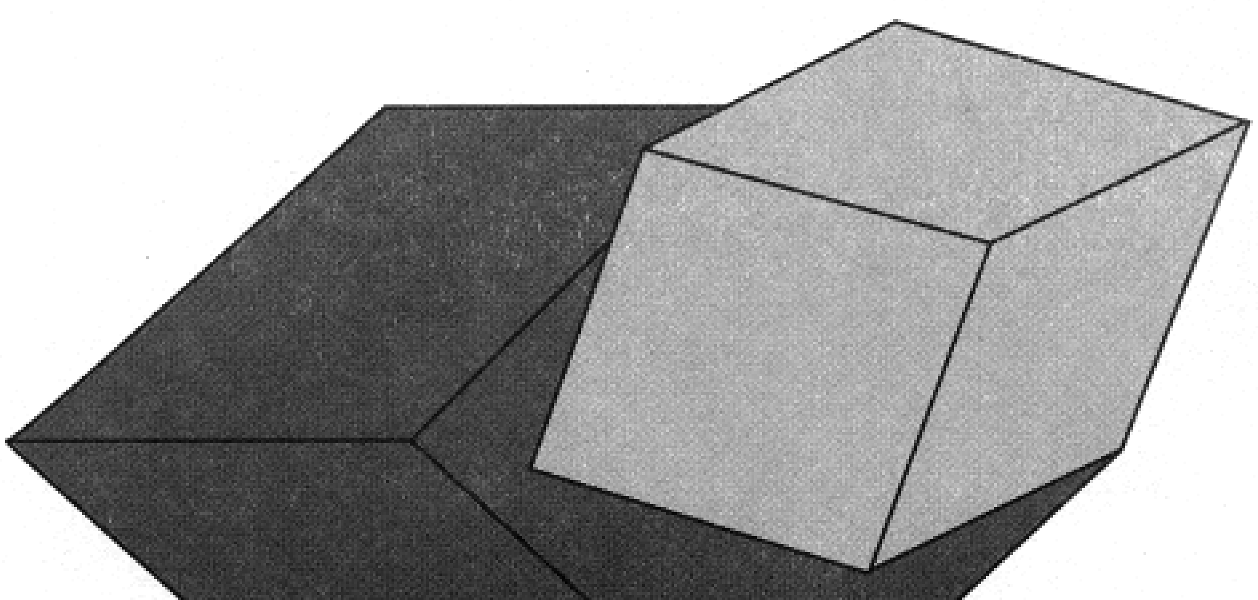

FORUM

BAICA'S EUCLIDEAN SOLUTION OF FERMAT'S LAST THEOREM (FLT)

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Abstract. The main interest of this paper is to put together all the information of the author's work on (FLT) that was given in her seven published papers. Also, we will answer some additional questions received after the publication of her paper titled *More Explanation about Baica's Proof of Fermat's Last Theorem* [4].

Key words[1]

Algorithm over the Complex Number Field	ACF
Baica's General Euclidean Algorithm	BGEA
Euclidean Algorithm	EA
Euclidean Geometry	EG
Euler Lagrange Theorem	ELT
Euler System	ES
n-Dimensional Euclidean Geometry	$\mathrm{E}^n\mathrm{G}$
2-Dimensional Euclidean Geometry	$\mathrm{E}^2\mathrm{G}$
Fermat's Last Theorem	FLT
Geometry of Elliptic Curves	GEC
Hasse and Bernstein Algorithm	HBA
Hyperbolic Geometry	HYG

In mathematics we can construct as many geometries or geometric models as we please. All that we need is to have the elements declared, to state the axioms and the definitions, and to have consistency in our mathematical logic. All of those many geometries do not report to each other, but they all report to the topology. Because of this, if you prove something in one geometry it may NOT be the same as in another geometry. Only one geometry is the Euclidean Geometry (EG), the other geometries are non–Euclidean Geometries. For example if we consider the Vth postulate in (EG) where two parallel lines do not intersect, they intersect at two ideal points Ω and Ω^1 in the Hyperbolic Geometry (HYG).

Since we cannot compute in a geometry, it is known that to every geometry we can associate a corresponding algebra but the converse is not true, and we

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call this algebra the number theory corresponding to that geometry. In every number theory there is a very strong theorem where if we implement rightly the conditions of other new theorems in the conditions of this main strong theorem, then those new theorems become immediate consequences of that initial strong theorem. That is known as the Euler System (ES) of that number theory. In other words, the (ES) in that corresponding number theory is a very powerful tool to prove many theorems in that number theory.

For example, the original (FLT) was stated in the (EG) and its corresponding number theory is known as the classical number theory. Since no "Geometry of the Elliptic Curves" (GEC) and its "Arithmetic of the Elliptic Curves" known as Hecke and Langlands Algebra to be the number theory of the (GEC) existed 350 years ago, we are forced to recognize the strong Euclidean character of (FLT) [3].

Initially the proof of (FLT) in the (GEC) was implemented wrongly in the (ES) and later it was abandoned in order to fix the gap. In this paper we show that we used (BGEA) restricted periodicity to be the (ES) for the algebraic number theory corresponding to the n-dimensional (EG) that is (E^nG), to prove (FLT) in Euclidean.

If the (ES) would not be abandoned in the proof of (FLT) in the (GEC), it could be known as having an equivalent result but not the same with our proof of (FLT) in (E^nG). It is very well known that in order to transfer the same result from one computing algebra system to another computing algebra system there is a need for an analytical continuous transformation (a functor) which is not only a simple transformation needed for an equivalent result. This anylitical continuous transformation is known to be the Galois connection needed to transfer a result from one category to the same result in another category that requires analytical continuity.

We will be tempted to recognize Falting's proof as an equivalent proof with our Euclidean proof since his commutative diagram with some minor modification will bring us to the solvability by radicals from Euclidean. Therefore, it may be shown that his proof of (FLT) in (GEC) is isomorphic with our proof of (FLT) in (E^nG) which is the original (FLT). It will be the same if the analytical continuous transformation will be provided. In [8] Faltings writes "the proof of the conjecture mentioned in the title was finally completed in September of 1994. A. Wiles announced this result in the Summer of 1993; however, there was a gap in his work. The paper of Taylor and Wiles does not close this gap but circumvents it. This article is an adaptation of several talks that I have given on this topic and is by no means about my own work. I have tried to present the basic ideas to a wider mathematical audience, and in the process I have skipped over certain details, which are in my opinion not so much of interest to the non–specialists. The specialists can then alleviate their boredom by finding those mistakes and correcting them".

One Euler's equation was wrongly named Pell's equation. This time we write this paper in order to avoid the same type of mistake regarding the proof of the original (FLT). Some people say the same thing about the name of the

Euclidean Algorithm. Who knows? It seems that the mathematics world is made to make lots of mistakes.

Solution of the problem

In [1] Baica uses a modification of (HBA) first time extended over the complex numbers and proved that $d|D$ is a necessary condition for her algorithm to be periodic at that time named (ACF).

I. In [2] she proved that $d|D$ is also a sufficient condition and proved completely the restricted periodicity of her algorithm now named (BGEA). In the same paper we identified her published papers which up to its restricted periodicity (BGEA) solved all the open questions in (E^n) which were solved in (E^2) as consequences from the always periodicity of the (EA) which is (ELT). Therefore we proved by generalization (FLT) also.

II. In [5] we showed that the (EA) is the (ES) for (E^2). This is the reply to "no proof confusion" of some professionals who criticize Baica's proof of (FLT) in the unprofessional publications. Baica's proof of (FLT) in Euclidean is a direct consequence of the restricted periodicity of her (BGEA) proof which is an n-dimensional equivalent of (ELT), and some mathematicians know that one does not prove a consequence of a theorem once the main theorem was proven.

III. In [4] we answered a question where we related the degree of the equation in the (FLT) with the dimension of (BGEA). It was Hilbert who related for the first time the degree of this equation $x^2 + y^2 = z^2$ with the always periodicity of the (EA). The dimension of the (EA) is given by the degree of the radical or irrational which makes it periodic. It shows that many mathematicians do not realize why Hilbert made this remark, and here I am coming with the answer. From the solvability by radicals a quadratic equation is solvable by a quadratic irrational in (E^2) and every quadratic irrational makes (EA) always periodic (ELT) where the dimension of the (EA) is $n = 2$ in the (BGEA).

IV. Many mathematicians keep calling wrongly Baica's General Euclidean Algorithm as my general continued fractions algorithm. It only happens that for $n = 2$ in (BGEA) which is (EA) can be identified with the continued fraction because (ELT) proves the always periodicity of the (EA) using the continued fractions algorithms. Jacobi and Perron used general continued fractions algorithm but they could not prove the periodicity or the restricted periodicity of their algorithm except for some numerical examples. The transformation in the continued fraction algorithm is the greatest integer function. The (HBA) was the closest algorithm to the (GEA) for reals and they did not use the greatest integer function as their transformation, but instead, they used the evaluation function as their transformation [7]. Baica used the same transformation as (HBA) for the first time over the complex numbers field and with this she proved an if and only if theorem for her (BGEA) restricted periodicity. I did not claim that my algorithm (BGEA) is anything else except the General Euclidean Algorithm.

Example I. The development of any quadratic W using (BGEA).

Let $a^{(0)} = W - D_2$ and $b^{(0)} = D_1 - D_2$ where $(W - D_1)(W - D_2) - d = 0$. Then by (BGEA)

$$a^{(1)} = d^{-1}(W - D_2);\ b^{(1)} = d^{(-1)}(D_1 - D_2);\ a^{(2)} = W - D_2 = a^{(0)}.$$

Here we have a purely periodic continued fraction representation for $W - D_2$ where

$$(W - D_1)(W - D_2) - d = 0;\ W - D_2 = \left[\,\overline{D_1 - D_2}\,\right] \text{ if } d = 1$$

$$W - D_2 = \left[\overline{(D_1 - D_2)\,d^{-1},(D_1 - D_2)}\right] : d > 1;\ d|D_1 - D_2$$

This is a very simple way of construction the periodic continued fraction of $\sqrt{3}$.

For $n = 2$; $(W - 4)(W - 2) - 2 = 0$; choose $W = 3 + \sqrt{3}$; $D_1 = 4$; $D_2 = 2$; $d = 2$; $K_1 = Q$; $K_2 = Q(\sqrt{3})$;

$$a^{(0)} = W - 2 = \left[\overline{\frac{4-2}{2}, 4-2}\right]$$

$$\sqrt{3} + 3 - 2 = [1,2];\ \sqrt{3} + 1 = \left[\,\overline{1,2}\,\right] \text{ and } \sqrt{3} = \left[\,1,\overline{1,2}\,\right]$$

Similarly for $\sqrt{2}$.

For $n = 2$; $(W-1)(W+1)-1 = 0$; choose $W = 1+\sqrt{2}$; $D_1 = 1$; $D_2 = -1$; $d = 1$;

$$K_1 = Q;\ K_2 = Q(\sqrt{2})$$

$$a^{(0)} = W + 1 = \sqrt{2} + 1 = \left[\,\overline{2}\,\right] \text{ and } \sqrt{2} = \left[\,1,\overline{2}\,\right]$$

Example II. The development of $\sqrt{2}$, and $\sqrt{3}$ by the continued fraction algorithm

For $a^{(0)} = \sqrt{3},\ b^{(0)} = 1$

$$a^{(1)} = \frac{1}{(\sqrt{3}-1)}\,\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{2},\ b^{(1)} = 1$$

$$a^{(2)} = \frac{1}{\dfrac{\sqrt{3}+1}{2} - 1} = \frac{2}{(\sqrt{3}-1)}\,\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \sqrt{3}+1,\ b^{(2)} = 2$$

$$a^{(3)} = \frac{1}{\sqrt{3}+1-2} = \frac{1}{(\sqrt{3}-1)}\,\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = a^{(1)}$$

$$\sqrt{3} = \left[b^{(0)}, \overline{b^{(1)}, b^{(2)}}\right] = \left[1, \overline{1,2}\right]$$

For $a^{(0)} = \sqrt{2},\ b^{(0)} = 1$

$$a^{(1)} = \frac{1}{(\sqrt{2}-1)}\frac{(\sqrt{2}+2)}{(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{1},\ b^{(1)} = 2$$

$$a^{(2)} = \frac{1}{\sqrt{2}+1-2} = \frac{1}{(\sqrt{2}-1)}\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{1} = a^{(1)}$$

$$\sqrt{2} = [1, \overline{2}\,]$$

Note. Only for $n = 2$; (BGEA) identify with the continued fraction because $n = 2$ in (BGEA) is the (EA), for $n \geq 3$ the restricted periodicity of (BGEA) is not proved by continued fractions and therefore it is not a generalization of the continued fractions algorithm, it is the generalization of the Euclidean Algorithm. It is true that sometimes some higher degree irrationals developed using (BGEA) when $d|D$ may coincide with the development using the continued fractions algorithm. There are far more irrationals which have (BGEA) periodic development than those who have periodic continued fractions development using the Jacobi Perron Algorithm.

V. Because of the confusion in IV, some mathematicians said that (BGEA) is not the (GEA) since the original Fibonacci numbers can be derived from the periodic expansion by the (EA) of $\sqrt{5}$ or by a periodic continued fraction development of $\sqrt{5}$ only. In the paper [6] the author opened a new horizon for the wanted generalization for the Fibonacci numbers and used (BGEA) restricted periodicity to derive n-dimensional Fibonacci numbers, and it first turns out that these n-dimensional Fibonacci numbers are most useful for a good approximation of algebraic irrationals by rational integers. For $n = 2$ in the n-dimensional Fibonacci numbers in [6] we obtain the original Fibonacci numbers.

VI. In [2] we claimed that the (BGEA) restricted periodicity proof gives the first algorithmic explicit proof for Hilbert's 10^{th} problem. There are complaints that there exists an explicit polynomial representation proof for Hilbert's 10^{th} problem. My answer to that is that this explicit polynomial representation is **not** an algorithmic explicit representation as (BGEA).

I know that this paper compliments everything which was published over my entire professional life in this subject, and therefore we have the proof of (FLT) in Euclidean. Fermat himself was so sure that he had the proof by induction but at that time he did not have the tool (BGEA) to make the induction on the dimension of (BGEA) and he got stuck after he proved his conjucture true for $n = 4$.

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