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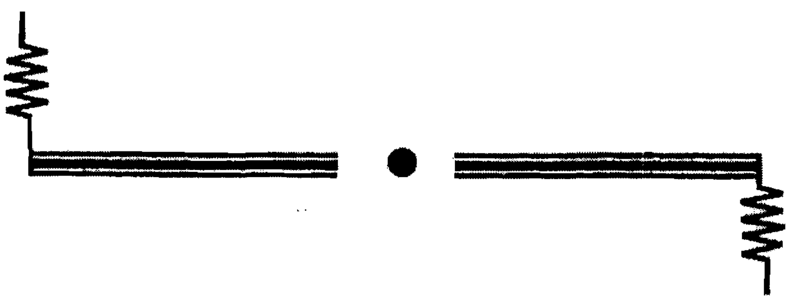

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PERIODIC TRANSTRIGONOMETRIC FUNCTIONS

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ABSTRACT

In this paper we analyze some basic functions with periodic character which can be considered as a new branch of Mathematics and we name it as "Transtrigonometry" (TT). This includes the "Quadratic trigonometry (QT)" and the "Classical trigonometry (CT)" as limiting cases. This Transtrigonomietry (TT) makes it possible to enlarge the application fields of Trigonometry to study the oscillation phenomenons.

KEYWORDS: Quadratic Trigonometry (QT), Classical Trigonometry (CT), Transtrigonometry (TT)

1 INTRODUCTION

It is known that many phenomenons in Physics and respectively in technical domains have an oscillation character. In many cases these phenomenons can be mathematically modelled with the help of the trigonometric functions *sin α* and *cos α* respectively. Example in this content are the unamortized mechanical vibrations [5], acoustic oscillations, electromagnetic waves etc.

There are some oscillation phenomenons of which mathematical representation does not have a sinusoidal form. In their analysis using the Classical trigonometry (CT), we apply the decomposition of these functions in Fourier series in order to do the mathematical modeling needed. Let give a single example in this regard, concerning line currents for the electrical transformer with a free current [2]. Intensity variation of a such current as a function of the period ωt is represented in Figure 1.

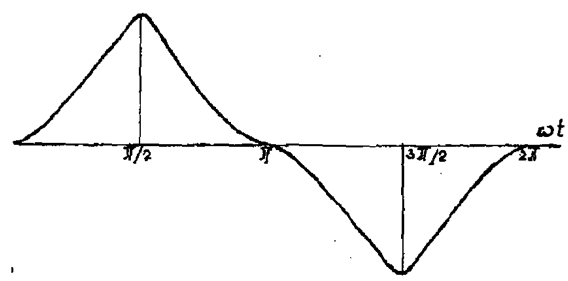

Figure 1

On the other side, in [2] and [4] the authors analyzed the bases of the Polygonal Trigonometry (PT) using the extended characteristic elements of QT [1].

2 TWO ESSENTIAL RELATIONS IN THE TRANSTRIGONOMETRY

As it is known the basic relations of CT and QT are the following:

In CT:

$$\sin^2 \alpha + \cos^2 \alpha = 1 \qquad (2.1)$$

In QT [1]:

$$sq\alpha + cq\alpha = 1 \qquad (2.2)$$

In PT [2,4]:

$$sp^k \alpha + cp^k \alpha = 1 \qquad (2.3)$$

where k has a variable value included in the domain $1<k<2$ [4].

It can be seen that the relations (2.2) and (2.3) are variable for the first trigonometric quadrant ($0 \le \alpha \le \pi/2$). In order that these relations to written under the form:

$$sq\alpha + cq\alpha = 1 \tag{2.4}$$

$$sp^k\alpha + cp^k\alpha = 1 \tag{2.5}$$

Relation (2.1) can be kept as it is since its valability from the algebraic point of view is preserved also for the negative values of *sinα* and *cosα* because they are raised to the second power.

On the basis of relations (2.1), (2.2) and (2.3) from above there appears in a logical way the idea to analyse some periodic functions of type *sin α*, *cos α* of CT which should satisfy a similar relation as (2.3) where k would have a constant value (not a variable on as in PT) and which should be included in the domain 1<k<2. At the lower neighbourhood of this domain (k=1) we have QT, and in the upper neighbourhood (k=2) we have CT.

We named Transtrigonometry (TT) the chapter of the Trigonometry which includes the domain between QT (k=1) and CT (k=2) and thus it is characterized by k=ct., having values in the domain 1<k<2. The functions of type "sinus α" respectively "cosinus α" we name them "Transtrigonometric sinus α" and "Transtrigonometric cosinus α" and we denote them with *st α* and *ct α*. In this way, similarly with the relations (2.4) and (2.5) we will have the relation

$$st\alpha^k + ct^k\alpha^k = 1 \tag{2.6}$$

Since k can have any value in the domain 1<k<2 in order to make distinctions between so many situations corresponding to various values of k in TT, we will characterize the functions *st α*, *ct α* etc by their "order" established by the value of k. The order will be denoted as index to *st α*, *ct α* etc as $st_k\alpha$, $ct_k\alpha$ etc.

Thus, to avoid confusions, relation (2.6) will be written as:

$$st_k\alpha^k + ct_k\alpha^k = 1 \tag{2.7}$$

It can be seen that the relation (2.7) has a general character, and relations (2.4) and (2.5) represent particular cases of that one. Thus, we can write sin α = st_2 α, cos α = ct_2 α and sq α = st_1α, cq α = ct_1α. In other words, the basic trigonometric functions of QT represent the respective functions "of first order" in TT. The same functions of CT represent the respective functions "of second order" of TT. Thus, the relation (2.7) is the first essential relation of TT.

On the other side, as we have shown in the papers [2] and [4], we can easily prove that the function "tangent" can be included in TT case in the equality:

$$tgt_k = tq\alpha = tg\alpha \tag{2.8}$$

The relation (2.8) represents the second essential relation of TT.

3 THE CHARACTERISTICS OF TRANSTRIGONOMETRIC FUNCTIONS

For the essential transtrigonometric functions, from (2.7) and (2.8) result the following expressions:

$$st_k\alpha = \pm\left(1 + ctg\alpha^k\right)^{-1/k} \tag{3.1}$$

$$ct_k\alpha = \pm\left(1 + tg\alpha^k\right)^{-1/k} \tag{3.2}$$

With relations (3.1) and (3.2) and knowing from CT the values for *tg* α and respectively *ctg* α we can compute the values for the functions st_k and ct_k α as functions of angle α, for diverse values of "order" k. The signs + (plus) and – (minus) in the front of formulas (3.1) and (3.2) – right sides – are given in function of the quadrant where angle α is situated. As in the CT, for α situated in I and II quadrants, $st_k\alpha$ has positive values and for α in III and IV quadrants, $st_k\alpha$ has negative values. On the other side, the function $ct_k\alpha$ has positive values in the quadrants I and IV, and negative values in the quadrants II and III.

In Figure 2 the function $st_k\alpha$ is represented for values of the angle α (expressed in radians) in the domain 0≤α≤2π, and for k=1 (QT), k=2 (CT) and k=1.4 (TT). We see that for k=2 the function "sinus" is represented by the classical sinusoid and for 1≤k≤2 the sinusoid curves have forms of "Arabian Archivolt" showing "fractures" for α = π/2 and α = 3π/2.

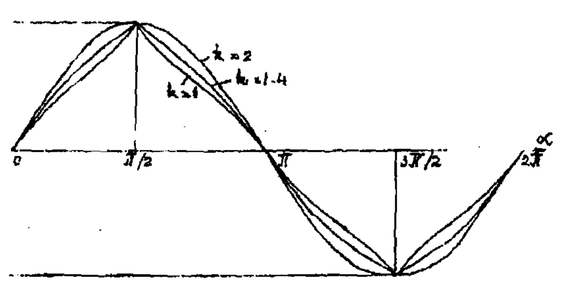

Figure 1

In Mathematics there is a distinction between the functions represented by monotonous curves and functions of type $st_k\alpha$ (for k=1 and k=1.4) represented by broken graphs consisting of several smooth arcs [5].

If in CT the functions *sinα* and *cosα* can be illustrated under geometrical form by referring to the "Trigonometric Circle" having a radius equal with the unity (R=1), in QT this is done for *sqα* and *cqα*, by referring to the "Trigonometric Square" [1] or better said "Trigonometric Rhombus" if we regard its position referring with the two orthogonal axis (horizontal and vertical). In its turn, the trigonometric rhombus is inscribed in a circle of R=1.

If we ask the question to determined the form of the Basic Geometric Figure equivalent with the Trigonometric Circle of CT and of the Rhombus (with straight sides) of QT, we see that this in x_k and y_k coordinates is represented by relation (2.7) making $st_k\alpha = x_k$ and $ct_k\alpha = y_k$. Thus we will have:

$$y_k^k + x_k^k = 1 \qquad (3.3)$$

and making y_k explicit we have:

$$y_k = \pm\left(1 + x_k^k\right)^{1/k} \qquad (3.4)$$

If we represent $y_k = f(x_k)$ in the domain $0 \leq \alpha \leq \pi/2$ (Quadrant I) for TT of order k=1.4, we will get the curve represented in Figure 3. It has the form of a "Curved Rhombus"(a rhombus with curved sides) and obviously, it is also inscribed in a circle of R=1.

In Figure 3, for comparison, we represented also the rhombus with straight sides (k=1) characteristic to QT and as much as the classical trigonometric circle (k=2) characteristic to CT, in quadrant I.

These last two figures also result in an analytic form if in relation (3.4) we introduce k=1 and k=2, respectively. The complete trigonometric rhombus, for k=1.4 is represented in Figure 4. The values for y_k will be positive (sign +) in relation $y_k = f(x_k)$ in I and II quadrants, and negative (sign -) in relation (3.4) for quadrants III and IV.

In Figure 3 we also find the elements with which we can easily repeat the proof for the relation (2.8). Thus, for similar right triangles OM'N', ONM and OM"N", we can write:

$$M'N' / O'N' = M"N" / ON" = MN / ON \qquad (3.5)$$

The trigonometric expression of relation (3.5) is represented by relation (2.8).

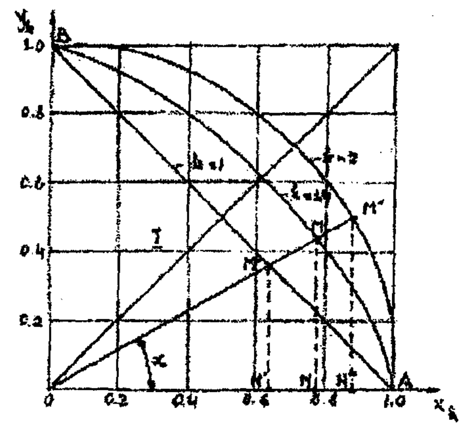

Figure 3

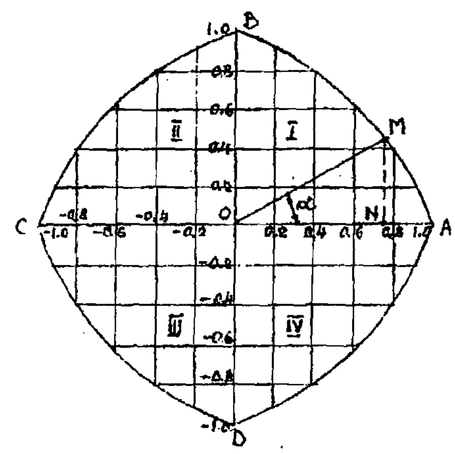

Figure 4

4 CONCLUSIONS

Transtrigonometry (TT) is a part of Trigonometry in which we study the periodic functions of type *sinus α*, *cosinus α*, etc which satisfy an essential relation more general then the relation (2.1) valid in the Classical Trigonometry (CT) and respectively the relation (2.4), valid in the Quadratic Trigonometry (QT).

In the TT the essential relation is the relation (2.7) where k=ct., comprised in the domain $1<k<2$. CT and QT are particular cases of TT.

The relation (28) is common for all three chapters of the Trigonometry (CT, QT and TT). The graphical form of the functions *sin α* and *cos* α (of CT) is a known continuous curve (the classical sinusoid), and the graphical form of the respective functions in QT and TT are "inscribed" in the classical sinusoid and they are different from it especially because of their "fractures".

Geometrically, these functions are characterised by "Arabian Archivolt" form.

Alike with the "Trigonometric Circle" of CT, in QT there is "Trigonometric Rhombus" with straight sides, and in TT the essential geometric figure is the "Trigonometric Rhombus" with curved sides.

Considering this aspect too, QT and CT represent limiting cases of TT.

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